(2) Maximize $Z = 3x_1 + 2x_2$, subject to the constraints

$x_1 - 3x_2 \le 3$; $x_1 - 2x_2 \le 4$; $2x_1 + x_2 \le 20$;

$x_1 + 3x_2 \le 30$; $-x_1 - x_2 \le 6$; $0 \le x_1 \le 8$ and $0 \le x_2 \le 6$.

Soln

$$Z - 3x_1 - 2x_2 = 0$$

$$x_1 - 3x_2 + S_1 = 3$$

$$x_1 - 2x_2 + S_2 = 4$$

$$2x_1 + x_2 + S_3 = 20$$

$$x_1 + 3x_2 + S_4 = 30$$

$$-x_1 - x_2 + S_5 = 6$$

number of non-basic Vars $= 7 - 5 = 2$

Let x_1 and x_2 are non basic Variables and

let $x_1 = x_2 = 0$ $\therefore$ $S_1 = 3$, $S_2 = 4$, $S_3 = 20$, $S_4 = 30$, $S_5 = 6$

P.C ↓

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	Soln	U_i
Z	-3	-2	0	0	0	0	0	0	-
S_1	1 (PE)	-3	1	0	0	0	0	3	∞
S_2	1	-2	0	1	0	0	0	4	∞
S_3	2	1	0	0	1	0	0	20	∞
S_4	1	3	0	0	0	1	0	30	∞
S_5	-1	-1	0	0	0	0	1	6	∞

x_1 entering Variable.

To find leaves Variable determine θ,

$\theta = \min\{\theta_1, \theta_2, U_1\}$ $\because$ x_1 entering Var $U_1 = 8$ upper bnd of x_1

$\theta_1 = \min\left\{\frac{\text{Soln}}{\text{+ve Coefft of pivot column}}\right\}$

$\theta_1 = \min\left\{\frac{3}{1}, \frac{4}{1}, \frac{20}{2}, \frac{30}{1}\right\} = 3$ Corr to S_1

$$\theta_2 = \left\{ \frac{u_i - x_{Bi}}{-(-\text{ve coeff of pivot column})} \right\}$$

$$\theta_2 = \left\{ \frac{\infty - 6}{-(-1)} \right\} = \infty \text{ corr to } S_5$$

$$\theta = \min\{\theta_1, \theta_2, u_1\} = \min\left\{ \overset{S_1}{3}, \overset{x_1}{8}, \overset{S_5}{\infty} \right\} = 3$$

corresponding to S_1

$\therefore$ S_1 leaves Variable.

↓ PC

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	Sln	u_i
Z	0	-11	3	0	0	0	0	9	~~u~~
New Pivot row → x_1	1	-3	1	0	0	0	0	3	8
S_2	0	1 PE	-1	1	0	0	0	1	∞
S_3	0	7	2	0	1	0	0	14	∞
S_4	0	6	-1	0	0	1	0	27	∞
S_5	0	-4	1	0	0	0	1	9	∞

Z-coeffs
$= -3+3(1) = 0$
$= -2+3(-3) = -11$
$= 0+3(1) = 3$
$= 0+3(0) = 0$
$= 0+3(0) = 0$
$= 0+3(0) = 0$
$= 0+3(0) = 0$
$= 0+3(3) = 9$

S_2-coeffs
$= 1-1(1) = 0$
$= -2-1(-3) = 1$
$= 0-1(1) = -1$
$= 1-1(0) = 1$
$= 0-1(0) = 0$
$= 0-1(0) = 0$
$= 0-1(0) = 0$
$= 4-1(3) = 1$

S_3-coeffs
$= 2-2(1) = 0$
$= 1-2(-3) = 7$
$= 0-2(1) = 2$
$= 0-2(0) = 0$
$= 1-2(0) = 1$
$= 0-2(0) = 0$
$= 0-2(0) = 0$
$= 20-2(3) = 14$

S_4
$= 1-1(1) = 0$
$= 3-1(-3) = 6$
$= 0-1(1) = -1$
$= 0-1(0) = 0$
$= 0-1(0) = 0$
$= 1-1(0) = 1$
$= 0-1(0) = 0$
$= 30-1(3) = 27$

S_5-coeffs
$= -1+1(1) = 0$
$= -1+1(-3) = -4$
$= 0+1(1) = 1$
$= 0+1(0) = 0$
$= 0+1(0) = 0$
$= 0+1(0) = 0$
$= 1+1(0) = 1$

x_2 entering Variable.

$$\theta = \min\{\theta_1, \theta_2, u_2\} \quad u_2 = 6$$

$$\theta_1 = \min\left\{ \frac{1}{1}, \frac{14}{7}, \frac{27}{6} \right\} = 1 \text{ corr to } S_2$$

$$\theta_2 = \min\left\{ \frac{8-3}{-(-3)}, \frac{\infty - 9}{-(-1)} \right\} = \min\left\{ \frac{5}{3}, \infty \right\} = \frac{5}{3} \text{ corr to } x$$

$\therefore \theta = \min\{\theta_1, \theta_2, u_2\}$

$\theta = \min\left\{1, \frac{5}{3}, 6\right\} = 1$ corr to S_2

$\therefore$ S_2 leaves Var.

PC ↓

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	Soln	u_i
Z	0	0	-8	11	0	0	0	20	
x_1	1	0	-2	3	0	0	0	6	8
New pivot row → x_2	0	1	-1	1	0	0	0	1	6
S_3	0	0	9 PE	-7	1	0	0	7	∞
S_4	0	0	7	-6	0	1	0	21	∞
S_5	0	0	-3	4	0	0	1	13	∞

Z-Coeffts

$= 0 + 11(0) = 0$

$= -11 + 11(1) = 0$

$= 3 + 11(-1) = -8$

$= 0 + 11(1) = 11$

$= 0 + 11(0) = 0$

$= 0 + 11(0) = 0$

$= 0 + 11(0) = 0$

$= 9 + 11(1) = 20$

x_1-Coeffts

$= 1 + 3(0) = 1$

$= -3 + 3(1) = 0$

$= 1 + 3(-1) = -2$

$= 0 + 3(1) = 3$

$= 0 + 3(0) = 0$

$= 0 + 3(0) = 0$

$= 0 + 3(0) = 0$

$= 3 + 3(1) = 6$

S_3-Coeffts

$= 0 - 7(0) = 0$

$= 7 - 7(1) = 0$

$= 2 - 7(-1) = 9$

$= 0 - 7(1) = -7$

$= 1 - 7(0) = 1$

$= 0 - 7(0) = 0$

$= 0 - 7(0) = 0$

$= 14 - 7(1) = 7$

S_4-Coeffts

$= 0 - 6(0) = 0$

$= 6 - 6(1) = 0$

$= -1 - 6(-1) = 7$

$= 0 - 6(1) = -6$

$= 0 - 6(0) = 0$

$= 1 - 6(0) = 1$

$= 0 - 6(0) = 0$

$= 27 - 6(1) = 21$

S_5-Coeffts

$= 0 + 4(0) = 0$

$= 4 + 4(1) = 0$

$= 1 + 4(-1) = -3$

$= 0 + 4(1) = 4$

$= 0 + 4(0) = 0$

$= 0 + 4(0) = 0$

$= 1 + 4(0) = 1$

$= 9 + 4(1) = 13$

S_1 entering Vari

To determine θ, $\theta = \min\{\theta_1, \theta_2$

$\theta_1 = \min\left\{\frac{7}{9}, \frac{21}{7}\right\} = \frac{7}{9}$ corr to S_3

$\theta_2 = \min\left\{\frac{8-6}{-(-2)}, \frac{6-1}{-(-1)}, \frac{\infty - 13}{-(-3)}\right\}$

$\theta_2 = \min\{1, 5, \infty\}$

$\theta_2 = 1$ corr to x_1

$\therefore \theta = \min\left\{\frac{7}{9}, 1, \infty\right\} = \frac{7}{9}$ corr to S_3

$\therefore S_3$ leaves Variable.

	x_1	x_2	S_1	S_2	S_3	S_4	S_5	soln	u_i
z	0	0	0	$\frac{43}{9}$	$\frac{8}{9}$	0	0	$\frac{236}{9}$	
x_1	1	0	0	$\frac{13}{9}$	$\frac{2}{9}$	0	0	$\frac{68}{9}$	
x_2	0	1	0	$\frac{2}{9}$	$\frac{1}{9}$	0	0	$\frac{16}{9}$	
New Pivot row → S_1	0	0	1	$-\frac{7}{9}$	$\frac{1}{9}$	0	0	$\frac{7}{9}$	
S_4									
S_5									

z-Coeffs

$= 0 + 8 \cdot 0 = 0$

$= 0 + 8 \cdot 0 = 0$

$= -8 + 8 \cdot 1 = 0$

$= 11 + 8\left(\frac{-7}{9}\right) = \frac{99-56}{9} = \frac{43}{9}$

$= 0 + 8\left(\frac{1}{9}\right) = \frac{8}{9}$

$= 0 + 8 \cdot 0 = 0$

$= 0 + 8 \cdot 0 = 0$

$= 20 + 8\left(\frac{7}{9}\right) = \frac{180+56}{9} = \frac{236}{9}$

x_1-Coeffs

$= 1 + 2(0) = 1$

$= 0 + 2(0) = 0$

$= -2 + 2(1) = 0$

$= 3 + 2\left(\frac{-7}{9}\right) = \frac{27-14}{9} = \frac{13}{9}$

$= 0 + 2\left(\frac{1}{9}\right) = \frac{2}{9}$

$= 0 + 2 \cdot 0 = 0$

$= 0 + 2 \cdot 0 = 0$

$= 6 + 2\left(\frac{7}{9}\right) = \frac{54+14}{9} = \frac{68}{9}$

x_2-Coeffs

$= 0 + 1(0) = 0$

$= 1 + 1(0) = 1$

$= -1 + 1(1) = 0$

$= 1 + 1\left(\frac{-7}{9}\right) = \frac{2}{9}$

$= 0 + 1\left(\frac{1}{9}\right) = \frac{1}{9}$

$= 0 + 1(0) = 0$

$= 0 + 1(0) = 0$

$= 1 + 1\left(\frac{7}{9}\right) = \frac{9+7}{9} = \frac{16}{9}$

Optimal Solution

$x_1 = \frac{68}{9}$

$x_2 = \frac{16}{9}$

$z = 3\left(\frac{68}{9}\right) + 2\left(\frac{16}{9}\right)$

$= \frac{204+32}{9} = \frac{236}{9}$